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Multivariate Surrogate Demand Modeling of Highway Bridge Structures

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ABSTRACT

Surrogate demand models (SDMs), which provide efficient estimation of engineering demand parameters (EDPs), are crucial components in probabilistic seismic risk assessment (PSRA) of structural systems. The accuracy and reliability of the resulting risk estimation are directly related to the predictive performance of the SDMs. For structural systems with multiple EDPs, multivariate surrogate demand modeling methods are desirable for their capability to handle the multicollinear EDPs simultaneously. However, such methods have not yet been widely applied in SRA. The current practice still mostly relies on developing univariate demand models separately for each EDP, requiring subsequent approximation of the correlation structure between the EDPs if at all considered. The present study conducts a comparative study of four different multivariate surrogate demand modeling approaches, namely Multivariate Linear Regression (MvLR), Linear Partial Least Squares Regression (L-PLSR), Kernel PLSR (K-PLSR) and Artificial Neural Network (ANN). To the authors' knowledge, this is the first time that PLSR is tested for surrogate demand modeling of structural systems. Two case-study highway bridges, including one multi-span simply-supported concrete girder (MSSS-Con) bridge and one multi-span continuous steel girder (MSC-Steel) bridge, are considered. Latin hypercube sampling is conducted to account for the uncertainties in bridge structural parameters. A hazard-consistent ground motion suite is compiled using the Conditional Mean Spectrum ground motion selection method and nonlinear time history analyses are conducted in OpenSees. According to the results, ANN and K-PLSR deliver the best predictive performance among all the compared multivariate SDMs. Particularly, K-PLSR reveals itself to be a very promising multivariate surrogate demand modeling approach for its superior predictive performance and stability, capability to handle small-size data with high-dimension feature space, computational efficiency, ease of model tuning, robustness to multicollinearity and good model transferability.

Keywords: seismic risk assessment, multivariate surrogate demand modeling, partial least squares regression, highway bridge structures, conditional mean spectrum ground motion selection

INTRODUCTION

In probabilistic seismic risk assessment (PSRA) of structural systems, statistical surrogate demand models (SDMs) are crucial components for efficient engineering demand parameter (EDP) estimation and uncertainty propagation. Often times, multiple EDPs in the same structure are of interest because they may be correlated to different failure modes. For example, for building structures, interstory drift ratio (IDR) and peak floor acceleration (PFA) are typically considered as the damage indicators for structural and non-structural components, respectively [1],[2]; for highway bridge structures, column drift ratio, deformations of bearings and abutments are important indicators of seismic damage of these bridge components [3],[4]. However, at present, the commonly adopted approach is to develop multiple separate univariate SDMs for each EDP [3]-[5], where the non-trivial model tuning and calibration has to be repeated multiple times and the correlations between different EDPs have to be approximated [4]. In this regard, multivariate regression methods, where multiple dependent variables can be handled simultaneously within the same model, are promising solutions. Moreover, multivariate regression methods may be more advantageous when the dependent variables are multicollinear [6],[7], which is often the case for the different EDPs within the same structure. To date, very few studies have adopted multivariate regression methods to model the correlated structural demands. Luco et al. [7] employed multivariate linear regression to develop SDMs for the interstory drift ratios at different stories for steel moment-resisting frame buildings. Goda and Tesfamariam [8] developed multivariate SDMs using copulas to model the maximum and residual IDR, and PFA at different story levels of a non-ductile reinforced concrete frame. Mangalathu et al. [9] applied artificial neural network in surrogate demand modeling of different structural components of highway bridges.

Among the candidates for multivariate regression, comparative analyses for SDMs have yet to be conducted, and notable prospects exist that have not been explored within the context of seismic surrogate demand modeling. Particularly, Partial Least

Squares Regression (PLSR) is a multivariate regression method to characterize the relations between a set of independent variables and one or more dependent variables. PLSR first extracts a set of mutually orthogonal latent variables (also known as the PLS components) by maximizing the covariance between the independent variables and the dependent variables. Then, the mapping between the resulting latent variables and the dependent variables are established through ordinary least squares regression. PLSR shares much in common with the Principle Component Regression (PCR) but identifies the latent variables in a supervised manner [10]. The ability to handle multiple dependent variables and high-dimension feature space (even when the number of independent variables exceeds the number of samples), model parsimony, robustness to predictor multicollinearity and computational efficiency [11]-[13] have made PLSR a standard tool in chemometrics. Also, there have been emerging applications of PLSR in many other scientific fields such as bioinformatics, medicine and social science.

In this study, we compare different multivariate regression methods, namely Multivariate Linear Regression (MvLR), Linear Partial Least Squares Regression (L-PLSR), Kernel Partial Least Squares Regression (K-PLSR) and Artificial Neural Network (ANN), in seismic surrogate demand modeling of structural systems. Two case-study highway bridge structures at a hypothetical site in Memphis, TN are considered. A hazard-consistent ground motion suite is selected using the Conditional Mean Spectrum method and a large number nonlinear time history analyses (NLTHAs) are carried out in OpenSees. The model predictive performance and stability, computational efficiency and other practical issues are evaluated and discussed.

MULTIVARIATE SURROGATE DEMAND MODELING OF HIGHWAY BRIDGES

Multivariate surrogate demand models establish the mapping between *p* predictors $\mathbf{X}=[X_1, X_2, ..., X_p]$ (e.g., ground motion intensity measures (IMs) and structural design parameters) and *m* dependent variables $\mathbf{Y}=[Y_1, Y_2, ..., Y_m]$ (e.g., different EDPs within the same structure). The theoretical background of the considered multivariate regression methods (MvLR, ANN and PLSR) is briefly introduced in this section.

Multivariate Linear Regression (MvLR)

MvLR seeks to find linear relations between **X** and **Y** through the following equation:

$$\mathbf{Y} = \mathbf{XB} + \mathbf{E} \tag{1}$$

where **Y** is a *n* by *m* matrix, **X** is a *n* by (*p*+1) matrix with an all one first column (considering the intercept terms), **B** is a (*p*+1) by *m* matrix of unknown parameters and **E** is a *n* by *m* regression residual matrix. *n* is the number of training samples. The least squares estimator of **B** can be obtained as follows [14]:

$$\hat{\mathbf{B}} = \left(\mathbf{X}^{\mathrm{T}}\mathbf{X}\right)^{-1}\mathbf{X}^{\mathrm{T}}\mathbf{Y} \tag{2}$$

Note that the regression coefficients $\hat{\mathbf{B}}$ in the above equation is equivalent to developing a suite of separate univariate linear regression models for each of the EDPs (also known as the widely-accepted Cloud analysis method by Cornell et al. [15] when only considering a single scalar IM as the predictor). However, the errors for the univariate linear regression models are assumed to be independent while the errors for the multivariate linear regression model are correlated.

Partial Least Squares Regression (PLSR)

PLSR establishes the mapping between the independent variables **X** and the dependent variables **Y** by means of mutually orthogonal latent variables (or PLS components). PLSR first performs matrix decomposition of the centered matrices **X** and **Y**:

$$\mathbf{X} = \mathbf{TP}^{\mathrm{T}} + \mathbf{E_X} \tag{3}$$

$$\mathbf{Y} = \mathbf{UQ}^{\mathrm{T}} + \mathbf{E_Y} \tag{4}$$

$$\mathbf{T} = \mathbf{XW} \text{ and } \mathbf{U} = \mathbf{YR} \tag{5}$$

where $\mathbf{T} = [\mathbf{t}_1, \mathbf{t}_2, \dots, \mathbf{t}_k]$ and $\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_k]$ are matrices of the extracted *k* PLS components of **X** and **Y**; **P** and **Q** represent the loading matrices; $\mathbf{E_X}$ and $\mathbf{E_Y}$ are the residual matrices; $\mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2, \dots, \mathbf{w}_k]$ and $\mathbf{R} = [\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_k]$ represent the weights of the PLS components of **X** and **Y**. The PLS components $\mathbf{t}_i$ and $\mathbf{u}_i$ as well as their corresponding weights $\mathbf{w}_i$ and $\mathbf{r}_i$ are successively extracted by maximizing the covariance between **X** and **Y**. Different component extraction algorithms have been proposed and the SIMPLS [12] algorithm is adopted herein for its high computational efficiency. Next, ordinary least squares regression is performed between the extracted PLS components **T** and **Y** as shown in Eq. (6). Note that the PLS components **T** are mutually orthogonal, which is favorable for least squares regression.

$$\mathbf{Y} = \mathbf{TC} + \mathbf{E} \tag{6}$$

The above equation can also be rewritten in terms of **X**:

$$\mathbf{Y} = \mathbf{XB} + \mathbf{E} \tag{7}$$

where $\mathbf{B} = \mathbf{WC}$. The optimal number of PLS components can be determined via cross validation. PLSR extracts the components in a supervised manner, thereby fewer components are required compared with Principle Component Regression (PCR). PLSR achieves dimension reduction and regression simultaneously and is found to be more robust to predictor multicollinearity [6],[11],[16]. PLSR can be further implemented in conjunction with a kernel transformation of the original **X** matrix:

$$K\left(\mathbf{x}_i, \mathbf{x}_j\right) = <\phi(\mathbf{x}_i), \phi(\mathbf{x}_j)> \tag{8}$$

where $K(\cdot)$ denotes the kernel transformation, $\mathbf{x}_i$, $\mathbf{x}_j$ are two generic rows of **X**, $\phi(\cdot)$ denotes a nonlinear mapping to a higher-dimension feature space, $< \cdot >$ denotes the inner product. By mapping the original feature space to a higher-dimension feature space, the potential nonlinear relations between the predictors can be better captured. In the present study, Kernel Partial Least Squares Regression (K-PLSR) with 2nd order and 3rd order polynomial kernels are considered. To distinguish from the K-PLSR method, the PLSR method using **X** without any kernel transformation will be referred to as the Linear PLSR (L-PSLR) method hereafter. For detailed derivation of PLSR, readers may refer to the following literature [11],[12],[17].

Artificial Neural Network (ANN)

The general structure of an ANN is shown in Figure 1. ANN is comprised of a collection of connected units (or neurons) associated to three types of layers: the input layer, the hidden layers and the output layer. In the present study, a single hidden layer is considered per Mangalathu et al. [9]. The nonlinear relations between the input **X** and the output **Y** are modeled through the connections between the neurons.

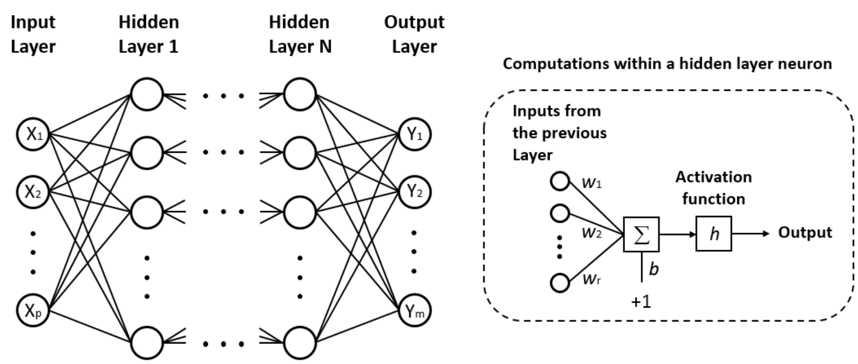

Figure 1. Schematic of artificial neural network structure

The output of a neuron in the hidden layer is a function of the linear combinations of the outputs from the neurons in the previous layer:

$$y = h\left(\sum w_i x_i + b\right) \tag{9}$$

where $h(\cdot)$ is the activation function (usually considered as the sigmoid function), w_i denotes the weights, x_i denotes the outputs from the previous later and b is a bias term. The network weights are first randomly initiated and are adjusted using the training set via a certain backpropagation algorithm. To control overfitting, a validation set is employed to determine when to stop the training process.

Estimation of the Error Covariance Matrix

In addition to the point estimates of the EDPs $\hat{\mathbf{Y}}$ obtained from the above mentioned multivariate regression models, the correlated errors of the EDPs should also be quantified for uncertainty propagation in PSRA. Assuming the errors follow a constant multivariate normally distribution, the maximum likelihood estimate of the error covariance matrix $\mathbf{\Sigma}$ is [14]:

$$\hat{\mathbf{\Sigma}} = \frac{\mathbf{E}^{\mathrm{T}}\mathbf{E}}{n} \tag{10}$$

where $\mathbf{E} = \mathbf{Y} - \hat{\mathbf{Y}}$ denotes the n by m regression residual matrix. The correlated errors can then be generated from the multivariate normal distribution $N(\mathbf{0}, \hat{\mathbf{\Sigma}})$.

CASE STUDY HIGHWAY BRIDGES AND GROUND MOTION SUITE SELECTION

Case Study Highway Bridges

Two highway bridges, one multi-span simply-supported concrete girder (MSSS-Con) bridge and one multi-span continuous steel girder (MSC-Steel) bridge, typical in the Central and Southeastern US region are considered for the case-study for comparative assessment of alternative multivariate SDMs. The two bridges are assumed to be located at a hypothetical site (-89.9, 35.2) in Memphis, TN. The bridge configurations are adopted from Nielson [4] as shown in Figure 2. The span length for the MSSS-Con bridge and MSC-Steel bridge is 24.4 m and 30.3 m, respectively. Both bridges have the same deck width of 15 m and deck slab depth of 0.178 m. Except for the given bridge geometric parameters as shown in Figure 2, uncertainties in other bridge structural parameters such as concrete and steel reinforcement strength, damping ratio, reinforcement ratio, abutment and foundation stiffness are considered via the Latin Hypercube Sampling technique. For each of the two case-study bridges, two datasets with 360 and 1000 bridge samples (approximately one and three times of the number of ground motion records selected (360) as shown in the next section) are generated to investigate the performance of the multivariate SDMs under different sample sizes. Each bridge sample is then randomly paired with a ground motion record. The parameterized OpenSees 3-D finite element models used in this study are adapted from Kameshwar et al. [18]. The sample median (T_{med}) of the geometric mean of the first two fundamental periods of MSSS-Con and MSC-Steel bridge is 0.56s and 0.33s, respectively. Spectral acceleration (Sa) at the period T_{med} is considered as the ground motion intensity measure (IM). The set of predictors considered for the subsequent surrogate demand modeling are listed in Table 1. Nonlinear time history analyses are performed and eight EDPs (as shown in Table 2) are recorded.

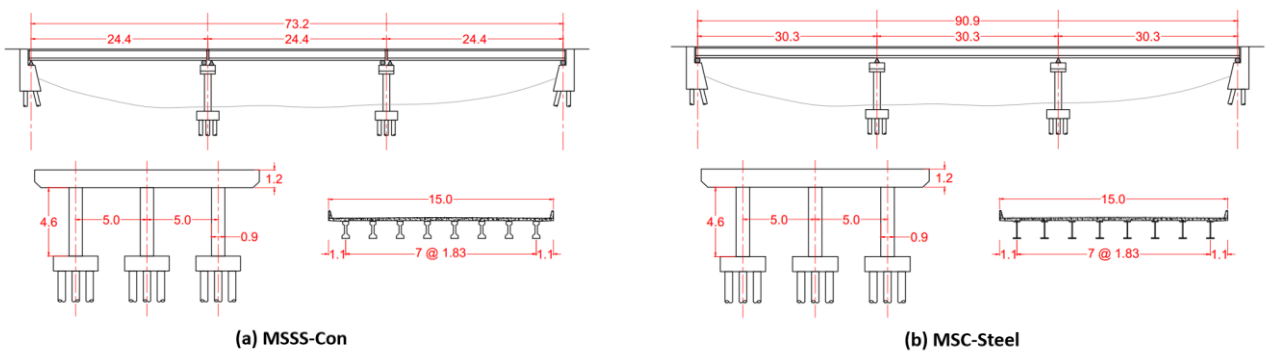

Figure 2. Schematics of the case study highway bridges (a) MSSS-Con bridge, (b) MSC-Steel bridge

*Table 1. Definitions of the predictors **X** for the two case-study bridges*

MSSS-Con		MSC-Steel	
Predictor	**Definition**	**Predictor**	**Definition**
X_1	Sa(0.56s)	X_1	Sa(0.33s)
X_2	Concrete strength	X_2	Concrete strength
X_3	Steel reinforcement yield strength	X_3	Steel reinforcement yield strength
X_4	Coefficient of friction of bearing pad	X_4	Abutment passive stiffness
X_5	Shear modulus of bearing pad	X_5	Abutment active stiffness
X_6	Dowel strength	X_6	Foundation vertical stiffness
X_7	Abutment passive stiffness	X_7	Foundation transverse stiffness
X_8	Abutment active stiffness	X_8	Mass participation ratio
X_9	Foundation vertical stiffness	X_9	Damping ratio
X_{10}	Foundation transverse stiffness	X_{10}	Column longitudinal reinforcement ratio
X_{11}	Mass participation ratio	X_{11}	Column transverse reinforcement ratio
X_{12}	Damping ratio	X_{12}	Coefficient of friction of high type steel fixed bearings - Longitudinal
X_{13}	Column longitudinal reinforcement ratio	X_{13}	Coefficient of friction of high type steel fixed bearings - Transverse
X_{14}	Column transverse reinforcement ratio	X_{14}	Coefficient of friction of high type steel expansion bearings - Longitudinal
		X_{15}	Coefficient of friction of high type steel expansion bearings - Transverse
		X_{16}	Steel bearing stiffness multiplication factor

*Table 2. Definitions of the EDPs **Y** for the two case-study bridges*

EDPs	Definition	Abbreviation
Y_1	Column drift ratio	Col
Y_2	Expansion bearing deformation - Longitudinal	Ebl
Y_3	Expansion bearing deformation - Transverse	Ebt
Y_4	Fixed bearing deformation - Longitudinal	Fbl
Y_5	Fixed bearing deformation - Longitudinal	Fbt
Y_6	Abutment deformation - Active	Aba
Y_7	Abutment deformation - Passive	Abp
Y_8	Abutment deformation - Transverse	Abt

Hazard-Consistent Ground Motion Suite Selection

To generate a ground motion suite consistent with the seismic hazard of the considered site, the conditional mean spectrum (CMS) ground motion selection method (originally proposed by Baker [19] and Jayaram et al. [20]) is employed. The general procedures of the CMS method are as follows: (1) Determine the primary conditioning IM (usually *Sa* at the fundamental period) target from the site-specific hazard curve and calculate the corresponding mean M, R and ε from deaggregation; (2) Derive the conditional multivariate normal distributions of all the IMs (usually *Sa* at a range of periods) considering the IM correlations; (3) Generate a number of target response spectra from the multivariate normal distribution obtained in (2); (4) Select the same number of ground motion records from a given database (e.g., PEER NGA-West 2 [21]) that best match the target spectra. Unlike ground motion selection methods based on the uniform hazard spectrum (UHS) which conservatively assumes constant risk at all the spectral periods, the CMS method can lead to more hazard-consistent ground motion records [19]. The original CMS method [19], [20] only considered *Sa* at a specific period as the primary conditioning IM and is hereafter referred to as CMS-*Sa*. Recently, Kohrangi et al. [22] proposed a modified version of the CMS method (CMS-*Savg*) by considering the spectral averaging IM *Savg* over a range of periods (as shown in Eq. (11)) as the primary conditioning IM.

$$Savg = \left(\prod_{i=1}^{N} Sa(T^{(i)}) \right)^{1/N} \tag{11}$$

Compared with CMS-*Sa*, CMS-*Savg* is able to select a suite of ground motions with further improved hazard consistency (i.e., lower scaling factor, better compatibility to multiple IMs and more moderate conditional variability across all spectral periods) [22]. Therefore, in this study, CMS-*Savg* ground motion selection method is employed considering the period range of 0.1-5.0s. Probabilistic seismic hazard analysis (PSHA) is conducted based on the 2008 USGS hazard model in OpenQuake. The ground motion prediction equations (GMPEs) for Eastern North America by Hassani and Atkinson [23] and the *Sa* correlation model by Baker and Jayaram [24] are adopted. In order to cover a wide range of ground motion intensities, 12 sets of scaled recorded ground motion records corresponding to return periods ranging from 30 to 2×10^5 years are selected for the site of interest. Each ground motion set contains 30 scaled ground motions, resulting in a total of 360 ground motion records. All of the records are selected from the PEER NGA-West2 database [21]. The conditional mean spectra along with the $\pm1\sigma$ lines (for the 12 return periods) as well as the response spectra of the individual selected ground motion records are shown in Figure 3.

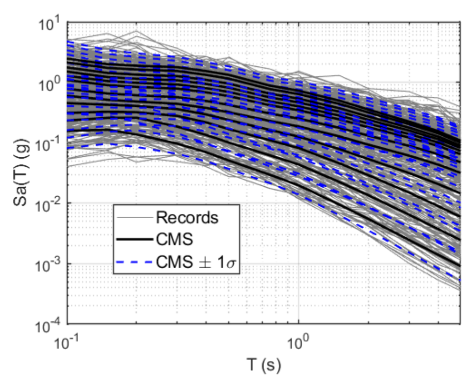

Figure 3. Conditional mean spectra and response spectra of the 360 selected ground motion records

RESULTS AND DISCUSSIONS

The predictive performance of different multivariate surrogate demand models is compared via repeated 5-fold cross validation. Note that for ANN, since a validation set is required in model training, a 60%-20%-20% data splitting is considered for cross validation. The cross validated mean squared error (MSE) is adopted as the goodness-of-fit metric. Note that for the ANN, L-PLSR and K-PLSR metamodels, all of the predictors in Table 1 are included and the best model tuning parameters are adopted according to experience from past literatures (e.g., only one hidden layer for ANN is considered as per Mangalathu et al. [9]), grid searching as well as trial and error. The optimal network structure and regularization parameter for ANN and the optimal number of PLS components for the PLSR models are listed in Table 3. For the MvLR metamodel, IM is considered as the sole predictor, which is in line with the practice of the conventional univariate Cloud analysis [15]. The predictors (**X**) and EDPs (**Y**) are log-transformed prior to the model fitting.

Table 3. Optimal model tuning parameters for different multivariate regression methods

Metamodels	**MSSS-Con**		**MSC-Steel**	
	360 samples	**1000 samples**	**360 samples**	**1000 samples**
L-PLSR	5	11	3	15
2nd order K-PLSR	17	16	19	19
3rd order K-PLSR	22	23	17	19
ANN*	3 neurons, reg = 0.001	5 neurons, reg = 0.001	3 neurons, reg = 0.1	5 neurons, reg = 0.001

* Levenberg-Marquartdt backpropagation algorithm is adopted

Metamodel Predictive Performance Comparison

The comparison of the cross-validated mean squared error (MSE) among different metamodels is shown in Table 4 to Table 7, for the two bridges and the two sample sizes. In general, the parameterized SDMs (L-PLSR, K-PLSR and ANN) are found to deliver much lower MSE level than the MvLR model. As mentioned earlier, in terms of the regression coefficients, MvLR is equivalent to a suite of separate univariate linear regression models (also known as the Cloud analysis as per Cornell et al. [15]), an approach that is widely adopted in the current practice. Therefore, the parameterized SDMs can provide more accurate seismic demand characterization due to the introduction of additional predictors and better capability to capture the nonlinear relations between the predictors and EDPs.

Among L-PLSR, K-PLSR and ANN, the L-PLSR model exhibits relatively inferior performance because the nonlinear interactions between the predictors are not accounted for. Further improvement is witnessed for the Kernel-PLSR (K-PLSR) with 2nd order or 3rd order polynomial kernels. We notice that K-PLSR with 2nd order kernel already delivers satisfactory performance whereas 3rd order kernel does not lead to much extra improvement. ANN and K-PLSR, with very close MSE levels, consistently outperform the other multivariate regression models. It should be noted that K-PLSR generally gives lower MSE standard deviation (values in parenthesis in Table 4 to Table 7), suggesting that the model predictive performance of K-PLSR is more stable than that of ANN. Moreover, ANN only shows slightly better predictive performance than K-PLSR when the sample size is relatively large (1000). For the smaller samples size (360), ANN fails to outperform the K-PLSR models. The fact that K-PLSR is able to better handle small sample size and large number of predictors can be beneficial in cases where each single run of NLTHA is so time consuming that one cannot afford to generate a large number of samples.

Table 4. Cross-validated MSE comparison for different multivariate regression methods (MSSS-Con, Sample size: 360)

Metamodels	**Col**	**Ebl**	**Ebt**	**Fbl**	**Fbt**	**Aba**	**Abp**	**Abt**	**Total (STD)**
MvLR	0.05	0.10	0.59	0.21	0.58	0.59	0.27	0.37	**2.75 (0.20)**
L-PLSR	0.05	0.08	0.56	0.17	0.55	0.27	0.26	0.17	**2.11 (0.16)**
2nd order K-PLSR	0.03	0.08	0.43	0.17	0.42	0.13	0.25	0.15	**1.65 (0.17)**
3rd order K-PLSR	0.03	0.08	0.43	0.16	0.42	0.13	0.26	0.14	**1.65 (0.16)**
ANN	0.05	0.10	0.40	0.15	0.39	0.16	0.26	0.16	**1.67 (0.24)**

Table 5. Cross-validated MSE comparison for different multivariate regression methods (MSSS-Con, Sample size: 1000)

Metamodels	**Col**	**Ebl**	**Ebt**	**Fbl**	**Fbt**	**Aba**	**Abp**	**Abt**	**Total (STD)**
MvLR	0.06	0.10	0.56	0.20	0.55	0.58	0.29	0.33	**2.66 (0.12)**
L-PLSR	0.05	0.09	0.53	0.17	0.53	0.26	0.26	0.16	**2.06 (0.09)**
2nd order K-PLSR	0.03	0.09	0.42	0.17	0.40	0.12	0.25	0.12	**1.61 (0.09)**
3rd order K-PLSR	0.03	0.09	0.42	0.15	0.40	0.12	0.24	0.12	**1.57 (0.09)**
ANN	0.04	0.09	0.38	0.14	0.36	0.12	0.24	0.12	**1.49 (0.12)**

Table 6. Cross-validated MSE comparison for different multivariate regression methods (MSC-Steel, Sample size: 360)

Metamodels	**Col**	**Ebl**	**Ebt**	**Fbl**	**Fbt**	**Aba**	**Abp**	**Abt**	**Total (STD)**
MvLR	0.16	0.34	0.62	1.27	0.65	0.21	2.51	0.30	**6.06 (0.53)**
L-PLSR	0.16	0.35	0.45	1.11	0.64	0.11	2.55	0.12	**5.50 (0.51)**
2nd order K-PLSR	0.14	0.34	0.37	1.10	0.46	0.10	2.48	0.09	**5.08 (0.66)**
3rd order K-PLSR	0.15	0.34	0.41	1.10	0.46	0.10	2.45	0.11	**5.12 (0.66)**
ANN	0.19	0.38	0.50	1.17	0.49	0.12	2.43	0.17	**5.44 (0.66)**

Table 7. Cross-validated MSE comparison for different multivariate regression methods (MSC-Steel, Sample size: 1000)

Metamodels	**Col**	**Ebl**	**Ebt**	**Fbl**	**Fbt**	**Aba**	**Abp**	**Abt**	**Total (STD)**
MvLR	0.17	0.33	0.68	1.07	0.66	0.19	2.46	0.30	**5.86 (0.33)**
L-PLSR	0.16	0.33	0.43	0.92	0.60	0.08	2.44	0.12	**5.10 (0.29)**
2nd order K-PLSR	0.13	0.30	0.34	0.86	0.43	0.08	2.27	0.10	**4.51 (0.29)**
3rd order K-PLSR	0.14	0.31	0.34	0.87	0.43	0.08	2.26	0.10	**4.52 (0.34)**
ANN	0.13	0.29	0.34	0.86	0.40	0.11	2.27	0.10	**4.51 (0.36)**

Discussions of Other Practical Issues

In this section, we will investigate and discuss some other practical advantages of PLSR. For the case of the 1000-sample MSSS-Con bridge, the comparison of computation time of different metamodels for 200 replicates of 5-fold cross validation is shown in Table 8. All of the computations were performed using Matlab R2017b [25] on a personal computer with an Intel Core i7-6700 CPU @ 3.40 GHz and 24 GB RAM. It can be seen that the L-PLSR and 2nd order K-PLSR are highly computationally efficient and they even outperform the simple MvLR model. Due to further expansion of the feature space, an increased clock time for the 3rd order K-PLSR (21.8 s) is reported. Among all of the metamodels, ANN turns out to be the most time consuming one, with about 58 times the clock time of the 2nd order K-PLSR.

Table 8. Comparison of the computation time of different metamodels for 200 replicates of 5-fold cross validation

	MvLR	**ANN**	**L-PLSR**	**2nd order K-PLSR**	**3rd order K-PLSR**
Clock time (s)	4.6	81.8	0.9	1.9	21.8

For ANN, in order to achieve optimal predictive performance, a lot of effort is required in tuning the model parameters, such as the network structures (number of hidden layers and number of neurons in each hidden layer), training algorithm and algorithm-related parameters, and regularization parameters for overfitting control. Moreover, as shown in Table 3, for different datasets, the corresponding optimal ANN model parameters may also vary. In contrast, only one tuning parameter (the number of PLS components) is required for PLSR. Therefore, compared with ANN, PLSR is not only computationally more efficient but also requires much less effort in model tuning. Additionally, like the conventional linear regression methods, PLSR embodies good model transferability, because closed-form equations can be easily obtained. Though not shown here, past studies [6],[11],[16] also revealed that PLSR is robust to multicollinearity in the predictors, which may be beneficial when correlated predictors (e.g., correlated structural parameters or vector IMs) are considered.

CONCLUSIONS

In order to provide more accurate estimation of multiple correlated engineering demand parameters in probabilistic seismic risk assessment of structural systems, different multivariate surrogate demand modeling approaches, including Multivariate Linear Regression (MLR), Linear Partial Least Squares Regression (L-PLSR), Kernel PLSR (K-PLSR) and Artificial Neural Network (ANN) are compared based on two typical highway bridge structures. Cross-validated mean squared error is considered to evaluate the predictive performance of different multivariate metamodels. According to the results, ANN and K-PLSR deliver the best predictive performance among all the multivariate regression methods compared. Particularly, K-PLSR reveals itself to be a promising alternative for multivariate surrogate demand modeling for its predictive performance stability, capability to handle small-size data with a high dimensional feature space, computational efficiency, ease of model tuning, robustness to multicollinearity and good model transferability. Future study should continue to explore the efficacy of PLSR in seismic surrogate demand modeling of other structural systems or a portfolio of structures as well as when a vector of multicollinear IMs are considered.

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